Transcomputation

Dr James Anderson FBCS CITP CSci
安德生

Agenda

- Discrete Logic
- Sets and Antinomies
- Physcial conformance of knowledge

Warning

- Although all of the information in this lecture has been discussed in scientific meetings, it has not all been published so there may be serious mistakes in it

Logic

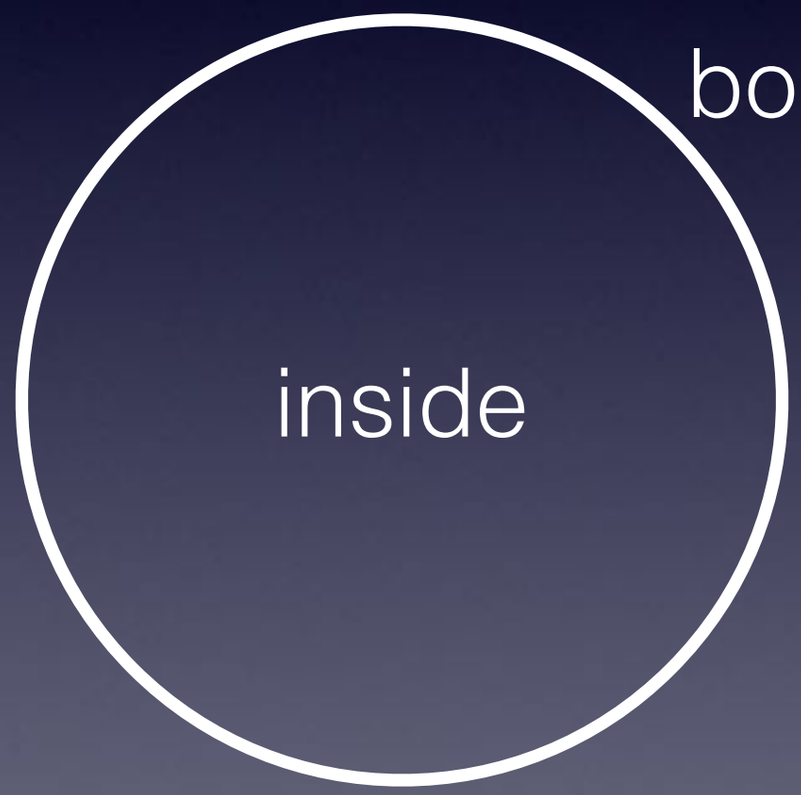
Logic

- What are all the combinations of the symbols $\{F\ T\}$?

Logic

- $\{F\}$ meaning False, F
- $\{T\}$ meaning True, T
- $\{F\ T\}$ meaning Contradiction, C
- $\{\}$ meaning Gap, G

Inside



Puzzle

- Sketch all of the points where it is true that they are inside the circle
- Sketch all of the points where it is false that they are inside the circle
- What is the truth value of the boundary?
- What things have the gap truth value and where are they?

Not

¬	F	T	C	G
	T	F	C	G

And

&	F	T	C	G
F	F	F	C	G
T	F	T	C	G
C	C	C	C	G
G	G	G	G	G

Logical connectives

- All Boolean logical connectives can be obtained from combinations of Not and And

- All digital electronic circuits can be obtained from NAND (Not And) gates

- What happens if we totalise Boolean logic?

Sets and antinomies

Naive-Set Theory

- The set $\{x \mid \phi(x)\}$ is defined by the class $\phi(x)$

- But classes are more general than sets so we are not surprised that naive set-theory is inconsistent

- The solution is to accept all of the notation and operations of naive set-theory as a class theory

Class Theory

- $\{x \mid \phi(x)\}$ is syntactic sugar for the class $\phi(x)$
- All other notations and operations of set theory are now generalised to classes

Interchangeability

- Interchangeability ($\doteq$) is an equivalence over all classes: $x \doteq y \Leftrightarrow x \in \{y\}$

Universes

- Universal Class (U): $U \doteq \{x \mid T\}$
- Universal Set (V): $V = \{x \mid x = x\}$
- Universal Antinomy (W): $W \doteq \{x \mid x \neq x\}$
- U is partitioned by V and W

Russell Class

- $R_U \doteq \{x_1 \mid x_2 \notin x_3\}$ with $x_1 \doteq x_2 \doteq x_3$

Russell Class

$$R_U \doteq \{x_1 \mid x_2 \notin x_3\}$$

- Suppose $R_U \doteq x_1$
- That is $R_U \in R_U$
- But $R_U \notin R_U$ because $x_2 \notin x_3$
- So $R_U \in R_U \Longrightarrow R_U \notin R_U$

Russell Class

$$R_U \doteq \{x_1 \mid x_2 \notin x_3\}$$

- Suppose $R_U \doteq x_2 \doteq x_3$
- That is $R_U \notin R_U$
- But $R_U \in R_U$ because x_1
- So $R_U \notin R_U \Longrightarrow R_U \in R_U$

Russell Paradox

- Combining implications we have a bi-implication that is the classical Russell Paradox:

- $R_U \in R_U \Leftrightarrow R_U \notin R_U$

- Russell assumed that the contradiction in the paradox proves that the set does not exist but it does exist, for us, as a class and we can work out its properties

Extensionality

- Axiom of Extensionality:

$$(x = y) \Rightarrow (z \in x \Rightarrow z \in y)$$

- Taking $x = y = z = R_U$
- Gives $(R_U = R_U) \Rightarrow (R_U \in R_U \Rightarrow R_U \in R_U)$
- But $R_U \in R_U \Leftrightarrow R_U \notin R_U$
- So $R_U \neq R_U$

Russell Antinomy

- $R_W \doteq R_U$

Russell Set

- $R_V = R_W \cap V$

Russell Truisms

- $R_W \in R_W$ is a gap
- $R_W \notin R_V$
- $R_V \in R_W$
- $R_V \notin R_V$

Classes

- Classes are defined in terms of continuous objects from which discrete objects can be drawn
- Objects can be used as symbols so that classes can, theoretically, be written in a continuous language
- But the physical properties of our universe limit what can be written
- Is our universe discrete or continuous?

Knowledge

Knowledge

- Knowledge can be expressed in digital or discrete systems
- But knowledge is embedded in the physcial universe so must conform to physics
- Von Neumann computers make the physically impossible assumption that information can travel faster than light
- Hypothesis: the more direct the conformance, the more efficient the computation

Reading

- Transreal arithmetisation of logic

- Trans-Boolean logic

Conclusion

- We need a total logic that covers at least the concepts: True, False, Contradiction, Gap

- We need to work with at least classes of both sets and antinomies, not just sets

- Knowledge may be discrete or continuous but it must conform to the physical arrangement of the universe